



***Life Cycle Assessment***  
***A product-oriented method***  
***for sustainability analysis***

***UNEP LCA Training Kit***  
***Module i – The mathematics of LCIA***



**UNEP**

Life Cycle



Initiative

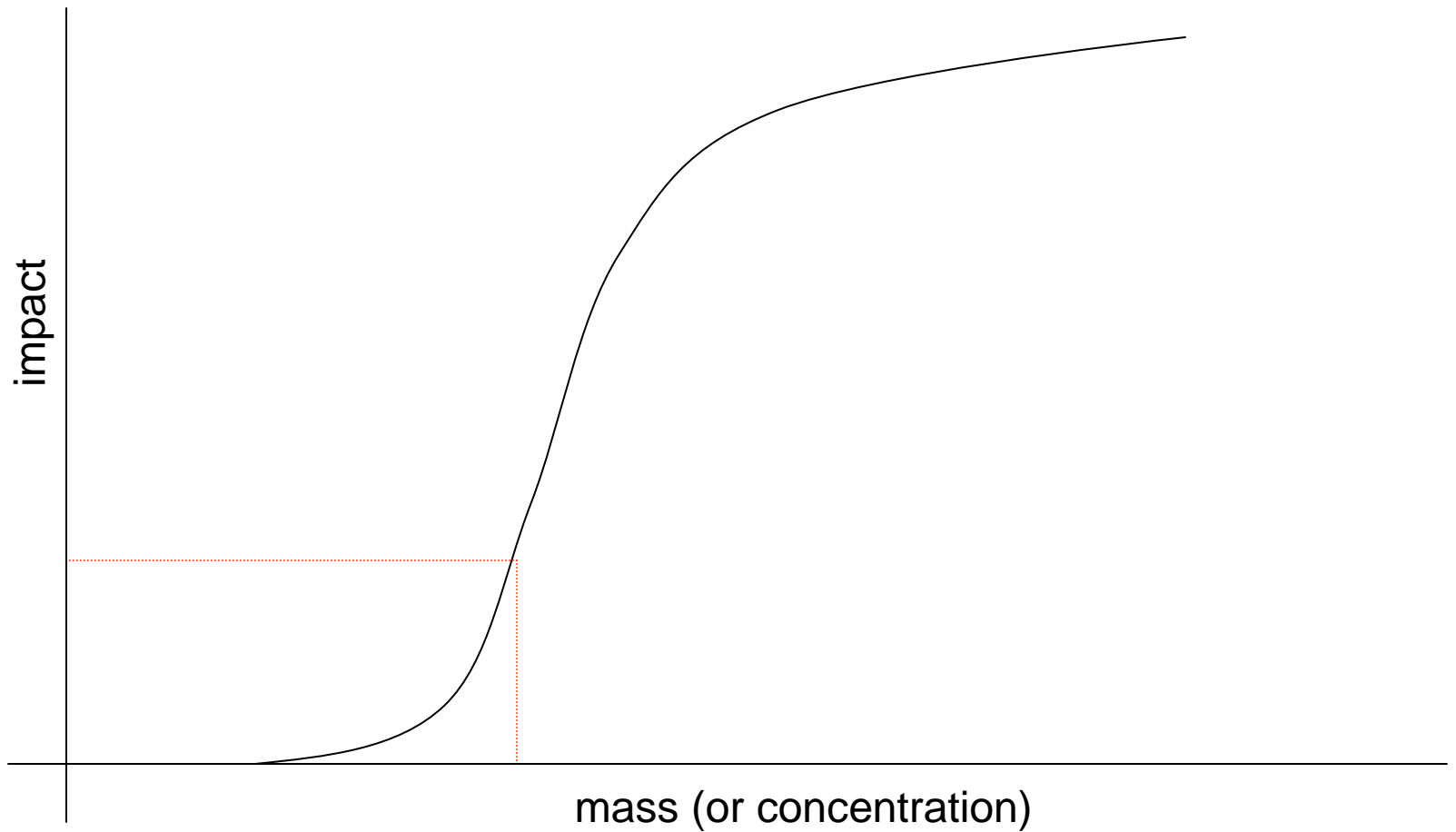


# Contents

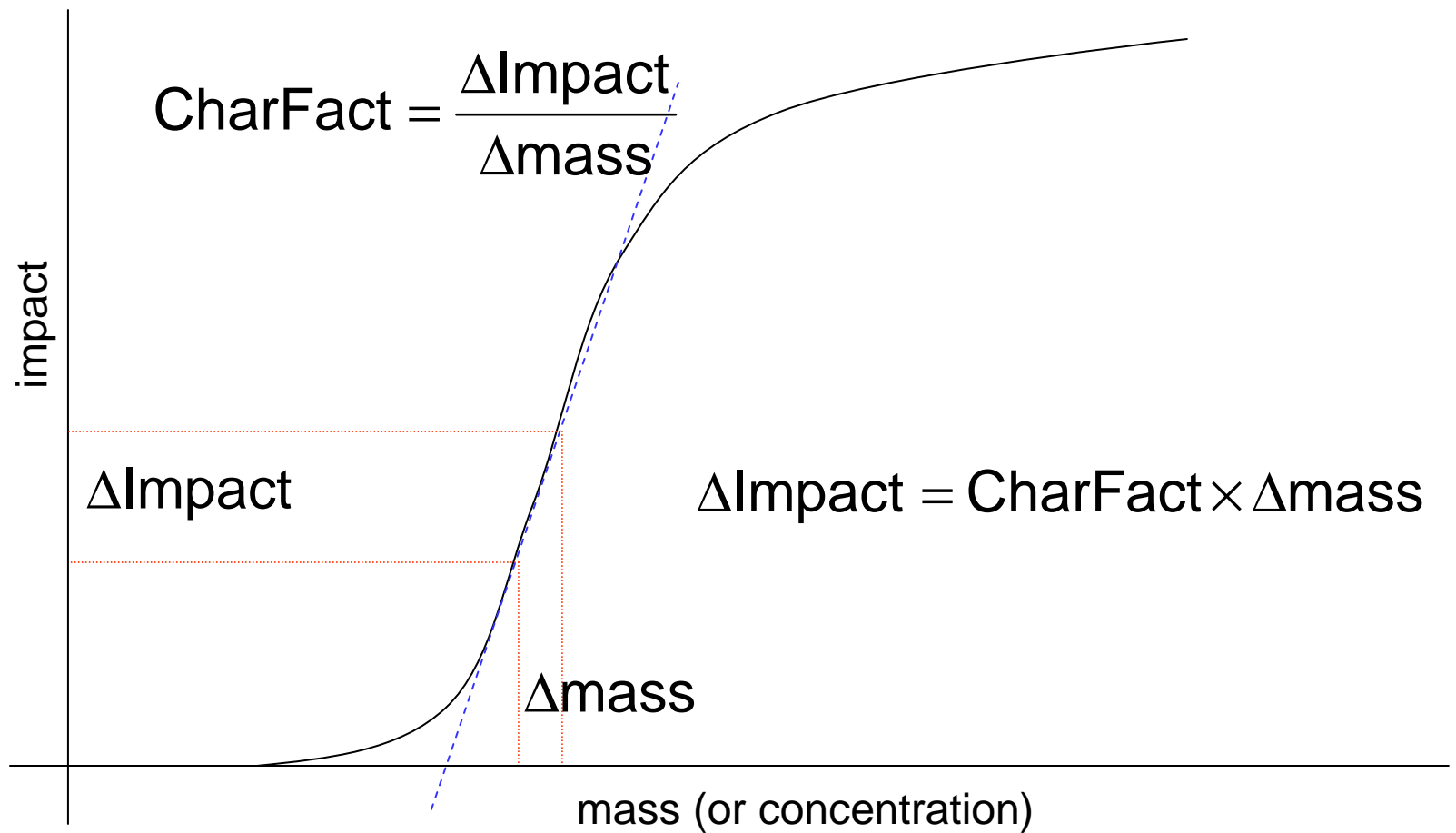
This module requires the delegate to have basic understanding of statistics and matrix algebra.

- **Characterisation factors**
- **Characterisation models**
- **The characterisation step**

# Characterisation factors



# Characterisation factors



# Characterisation factors

- **Infinitesimal small change:**

$$\text{CharFact} = \frac{\partial \text{Impact}}{\partial \text{mass}}$$

- **Substance and category dependence:**

$$\text{CharFact}_{cat,subs} = \frac{\partial \text{Impact}_{cat}(subs)}{\partial \text{mass}_{subs}}$$

- **Multiple substance dependence**

$$\text{CharFact}_{cat,subs} = \frac{\partial \text{Impact}(subs = 1, subs = 2, \dots)}{\partial \text{mass}_{subs}}$$

# Characterisation models

- **Characterization models express the relationship between the category indicator and the outputs.**

Impact = CharacterisationModel(subs = 1, subs = 2)

- **Characterization factors are derived from models that predict the impact.**

$$\text{CharFact}_{cat,subs} = \frac{\partial \text{CharModel}(subs = 1, subs = 2, \dots)}{\partial \text{mass}_{subs}}$$

# Characterisation models

- **Characterisation models are often developed outside LCA.**
  - International Panel on Climate Change (IPCC), World Meteorological Organisation (WMO), ...
- **They can be derivative.**
  - linearised model
  - only valid for small change.
- **They can be a partial derivative.**
  - constant background

# The characterisation step

- **Conversion and aggregation**

- conversion

$$\text{IndicatorResult}_{cat,subs} = \text{CharFact}_{cat,subs} \times \text{InventoryResult}_{subs}$$

- aggregation

$$\text{IndicatorResult}_{cat} = \sum_{subs} \text{IndicatorResult}_{cat,subs}$$

- together

$$\text{IndicatorResult}_{cat} = \sum_{subs} \text{CharFact}_{cat,subs} \times \text{InventoryResult}_{subs}$$



# The characterisation step

- or in matrix form

$$\begin{pmatrix} \text{IndicatorResult}_{cat=1} \\ \text{IndicatorResult}_{cat=2} \\ \dots \end{pmatrix} = \begin{pmatrix} \text{CharFact}_{cat=1,subs=1} & \text{CharFact}_{cat=1,subs=2} & \dots \\ \text{CharFact}_{cat=2,subs=1} & \text{CharFact}_{cat=2,subs=2} & \dots \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \text{InventoryResult}_{subs=1} \\ \text{InventoryResult}_{subs=2} \\ \dots \end{pmatrix}$$

- or even more condensed

$$\mathbf{h} = \mathbf{Qg}$$

- There are no problems with inversion or non-square matrices.



- **Characterisation factors**
- **Characterisation models**
- **The characterisation step**



The remaining modules  
explore these topics:

<b>Module</b>	<b>contents</b>
j	Life cycle costing
k	Uncertainty in LCA
l	Carbon footprint