Life Cycle Assessment

A product-oriented method for sustainability analysis

UNEP LCA Training Kit

*Module h – **The mathematics of LCI***

SETAC

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Life Cycle Initiative

Contents

This module requires the delegate to have basic understanding of matrix algebra.

- **Representation of processes**
- **Representation of goal definition**
- **Balancing**
- **Environmental interventions**
- **Overall LCI structure**
- **Advantages**
- **New concepts**

Representation of processes

- **Production of electricity**:
  - expressed in flow diagram terms:

2 liters of fuel →  → 10 kWh of electricity

→ 1 kg of CO_2

→ 0.1 kg of SO_2

  - expressed in mathematical terms:

$$\begin{pmatrix} -2 \\ 10 \\ 1 \\ 0.1 \\ 0 \end{pmatrix}$$

Representation of processes

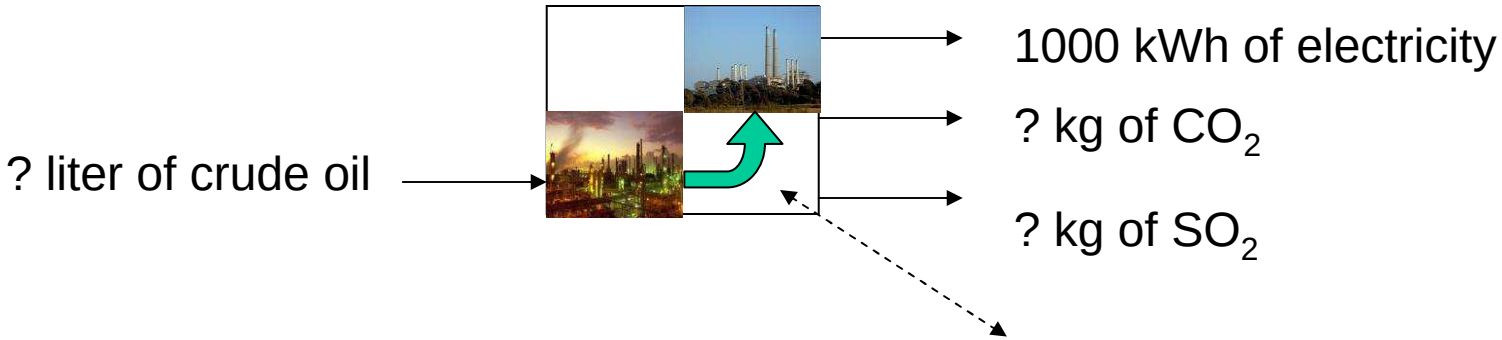
- **Production of fuel**:
  - expressed in flow diagram terms:

  - expressed in mathematical terms:

$$\begin{pmatrix} 100 \\ 0 \\ 10 \\ 2 \\ -50 \end{pmatrix}$$

Representation of goal definition

- **Functional unit/reference flow**:
  - expressed in flow diagram terms:

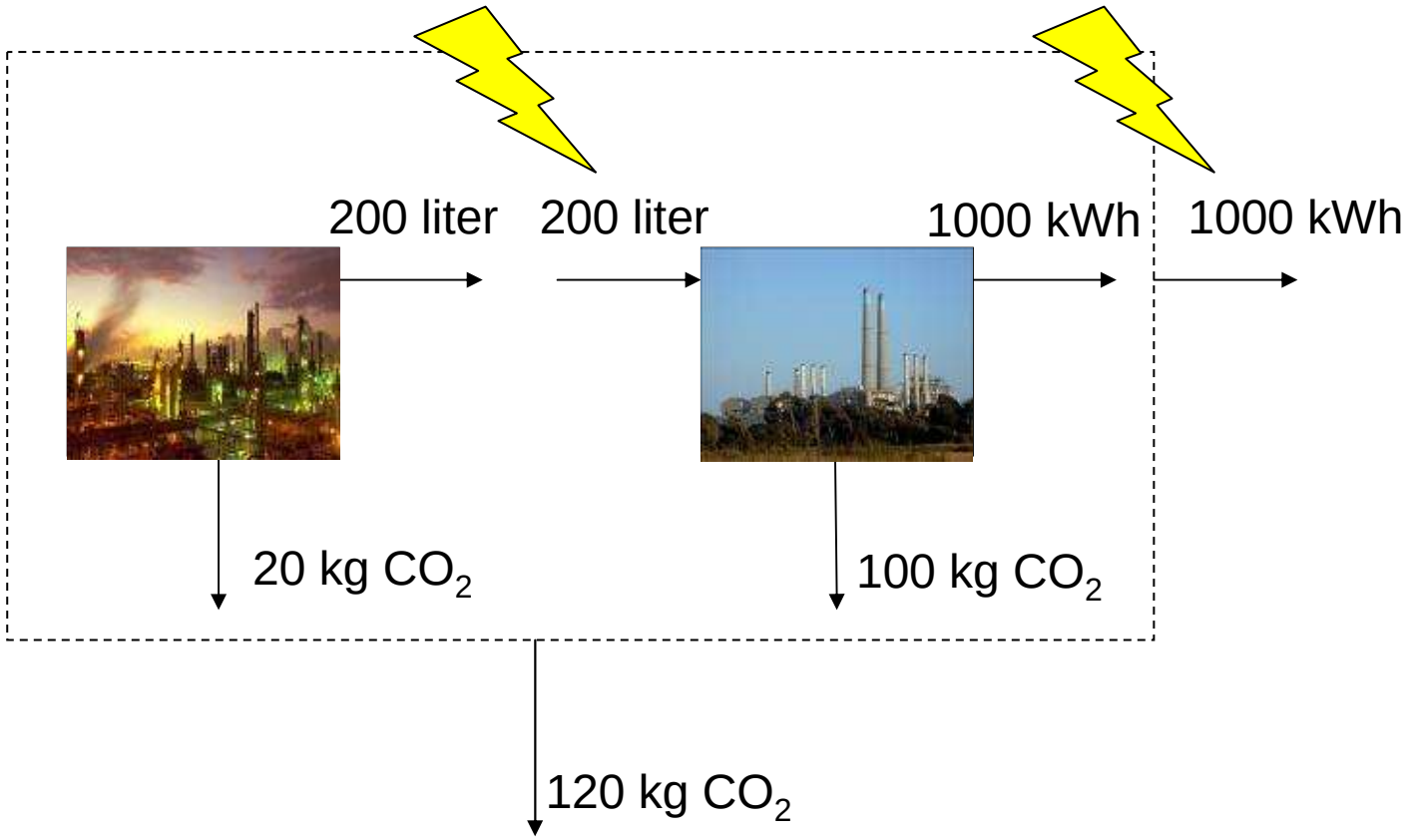


  - expressed in mathematical terms:

$$\begin{pmatrix} 0 \\ 1000 \\ ? \\ ? \\ ? \end{pmatrix}$$

Balancing

- **We need to match supply and demand**.



Balancing

- **We need to scale processes**:

$$\begin{cases} -2 \times s_1 + 100 \times s_2 = 0 \\ 10 \times s_1 + 0 \times s_2 = 1000 \end{cases}$$

$$\begin{pmatrix} -2 & 100 \\ 10 & 0 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1000 \end{pmatrix}$$

Balancing

- **From concrete equations with matrix coefficients**

$$\begin{pmatrix} -2 & 100 \\ 10 & 0 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1000 \end{pmatrix}$$

- **… to abstract equations with symbols**

$$\mathbf{As} = \mathbf{f}$$

- **… that can be solved by standard techniques**

$$\mathbf{s} = \mathbf{A}^{-1}\mathbf{f}$$

- **… and becoming concrete again**

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 2 \end{pmatrix}$$

Environmental interventions

- **Applying the same scaling factors to environmental flows:**

$$\begin{cases} 1\times s_1 + 10\times s_2 = ? \\ 0.1\times s_1 + 2\times s_2 = ? \\ 0\times s_1 + -50\times s_2 = ? \end{cases}$$

- **In matrix terms:**

$$\begin{pmatrix} 1 & 10 \\ 0.1 & 2 \\ 0 & -50 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix}$$

Environmental interventions

- **From concrete equations with matrix coefficients**

$$\begin{pmatrix} 1 & 10 \\ 0.1 & 2 \\ 0 & -50 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix}$$

- **to abstract equations with symbols**

$$\mathbf{Bs} = \mathbf{g}$$

- **in which we can insert the previous result**

$$\mathbf{g} = \mathbf{BA}^{-1}\mathbf{f}$$

- **and becoming concrete again.**

$$\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} 120 \\ 14 \\ -100 \end{pmatrix}$$

Overall LCI structure

- **Combining balancing and interventions:**

$$\mathbf{g} = \mathbf{B}\mathbf{A}^{-1}\mathbf{f} = \mathbf{\Lambda}\mathbf{f}$$

- **with the intensity matrix**

$$\mathbf{\Lambda} = \mathbf{B}\mathbf{A}^{-1}$$

Advantages of mathematical treatment

- **The quantitative method allows explicit treatment, and is transparent.**
- **It provides guidance for software implementation.**
- **It offers new insight in LCA.**
  - e.g., allocation
- **There are also advanced theoretical concepts:**
  - perturbation theory
  - covariance structure
  - stochastic theory

New concepts

- **Terms:**
  - technology matrix, scaling vector, intervention matrix, final demand vector, intensity matrix, …
- **Techniques:**
  - matrix conditioning, principal components analysis, …

You may wish to review some segments of this module on LCA mathematics.

- **Representation of processes**
- **Representation of goal definition**
- **Balancing**
- **Environmental interventions**
- **Overall LCI structure**
- **Advantages**
- **New concepts**

The remaining modules explore these topics:

Module	contents
i	LCIA mathematics
j	Life cycle costing
k	Uncertainty in LCA
l	Carbon footprint